ISI – Bangalore Center – B Math - Physics III – Mid Term Exam
Date: 11 September 2014. Duration of Exam: 3 hours
Total marks: 35

Q 1 [Total Marks: 14]

Q 1a. A infinitesimally thin ring of charge with radius a , total charge Q and uniform charge density is in the x-y plane with its center at the origin. What is the charge density in cylindrical coordinates. (Derivation of expression need not be shown.) [2]

Q 1b. A charged spherical shell of uniform surface charge density is kept inside an uncharged conducting thick hollow spherical shell, just below the inner metallic surface but without touching it. Draw a picture CLEARLY showing charge distributions and densities present in this system [2]

Q1c. Continuing with the question in Q 1b, suppose that due to thermal contraction, the inner metallic surface comes into contact with the charged shell. The surfaces are still spherical. Draw a picture showing the charge distribution after the contact takes place and after static condition is restored [2]

Q1d. Compare the electrostatic energies of the systems in Q 1b and 1c. Are they the same and if not which one is more than the other? Explain (a detailed calculation is not necessary. [2]

Q1e. For a tightly wound infinite solenoid with n turns of wire per unit length carrying current *I,* write an expression for surface current density $\vec{K}$. Assume that the axis of the solenoid is the *z* axis. (Derivation of expression need not be shown.) [2]

Q1f. For a hollow cylindrical conductor of radius *a*, infinite length with its axis aligned with the z axis and carrying current *I* along its length write an expression for the surface current $\vec{K}$. (Derivation of expression need not be shown.) [2].

Q1g. Given a charge density $\rho(\vec{r}) = k$ where k is a constant, show that the following electric fields $\vec{E}_1 = k\vec{i}$ and $\vec{E}_2 = k\vec{j}$ are both solutions to Poisson equation. Explain in one or two sentences why the uniqueness theorems of electrostatics are not applicable in this case.[2]

Q2. [Total Marks: 11]

There are two thin concentric metallic cells of radius a and b ($b>a$). Inner shell carries a total charge Q and the outer shell carried a total charge $-Q$.

Q2a. Draw a graph of V(r) as a function of r showing clearly the values of V(0), V(a), V(b). Assume V goes to zero as r goes to infinity. [4]

Q2b. Calculate the total electrostatic energy stored in the system. [4]

Q2c. Calculate the capacitance of this system. [3]

Q3. [Total Marks: 10]

3a.) State without derivation boundary conditions which are satisfied by magnetic fields across a surface carrying a surface current . [3]

3b.) Verify these boundary conditions for the case of an infinitely long solenoid. You can use the expression for magnetic field for this case without deriving it. [3]

3c.) Let $\vec{A} = A_\rho \hat{\rho} + A_\varphi \hat{\varphi} + A_z \hat{z}$ denote the magnetic vector potential. Calculate A_φ for the infinitely long solenoid whose axis is along the z axis.[4]